Практика

Пример декодирования кода конкатенации

C_{out} – код $RS_{\mathbb{F}_{2^3}, \mathbb{F}_{2^3}^*}[7,3,5]$, $\mathbb{F}_{2^3} \simeq \mathbb{F}_2[x]/(x^3+x+1)$

$\mathbb{F}_{2^3}^* = \{1, \alpha, \alpha^2, \alpha+1, \alpha^2+\alpha, \alpha^2+\alpha+1, \alpha^2+1\}$

C_{in} - бинарный $[7,3,3]$ код с порождающей G и проверочной H:

$$G = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix} \qquad H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

ПАР-РЫ: $n_{concat} = n_{out} \cdot n_{in} = 49$; $k_{concat} = k_{out} \cdot k_{in} = 9$; $d_{concat} \geq d_{out} \cdot d_{in} = 15$

Декодировать $y = [(0001100), (1100001), (0011101), (0100101), (0000000), (0000000), (0011111)]$

1. Декодируем каждый $y_i \in \mathbb{F}_2^7$ $(y = (y_1, y_2 \dots y_7))$ с помощью таблицы синдромов для C_{in}

e	$H \cdot e$
$(1000000)^T$	$(1000)^T$
$(0100000)^T$	$(0100)^T$
$(0010000)^T$	$(0010)^T$
$(0001000)^T$	$(0001)^T$
$(0000100)^T$	$(0101)^T$
$(0000010)^T$	$(1100)^T$
$(0000001)^T$	(0110)

1.1 $y_1 = (0001100)$

$H \cdot y_1 = (0100)^T \Rightarrow e_1 \ (0100000) \Rightarrow$

$$\Rightarrow c_1 = y_1 + e_1 = (0100100) \Rightarrow m_1 = \begin{array}{ccc} 1 & \alpha & \alpha^2 \\ \end{array}$$
$$m_1 = [1 \ 0 \ 0]$$

$\Rightarrow m_1 = 1 \in \mathbb{F}_2[x]/(x^3+x+1)$

1.2. $y_2 = (1100001)$

$H \cdot y_2 = (1010)^T$

$\Rightarrow m_2 = "\times"$

1.3. $y_3 = (0011101)$

$H \cdot y_3 = (0000)^T \Rightarrow y_3 \in C_{in} \Rightarrow m_3 = \begin{array}{ccc} 1 & \alpha & \alpha^2 \\ [0 & 0 & 1] \end{array} \Rightarrow m_3 = \alpha^2$

1.4. $y_4 = (0100101)$

$H \cdot y_4 = (0111)^T \Rightarrow m_4 = \text{"x"}$

1.5 = 1.6. $y_5 = y_6 \in C_{in} \Rightarrow m_5 = m_6 = [0,0,0] \cong 0 \in \mathbb{F}_{2^3}[x]/(x^3+x+1)$

1.7. $y_7 = (0011111)$

$Hy_7 = (1100)^T \Rightarrow e_7 = (0000010) \Rightarrow c_7 = y_7 - e_7 = (0011101)$

$m_7 = [001] \rightarrow \alpha^2$

2. На вход C_{out} получает $[1, \text{"x"}, \alpha^2, \text{"x"}, 0, 0, \alpha^2]$

$\hookrightarrow$ lagrange_polynomial $\rightarrow f = (\alpha^2+\alpha+1)x^2 + (\alpha^2+\alpha+1)x + 1$

$\rightarrow m = [1, \alpha^2+\alpha+1, \alpha^2+\alpha+1]$