Advances and Open Challenges in Quantum Algorithms for Lattice Problems

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based on joint works with

Shi Bai Zvika Brakerski Hansraj Jangir Tran Ngo
Damien Stehlé Weiqiang Wen William Youmans

at PQCrypto 2026

Part I

Learning With Errors Quantumly

The Dihedral Group D_{2N}

is a group of rotations and reflections of a regular $N-$gon.

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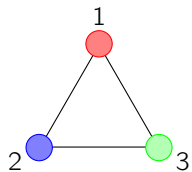

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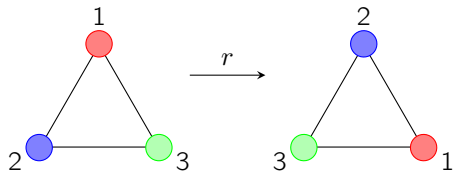

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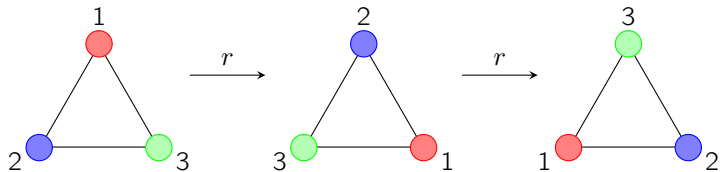

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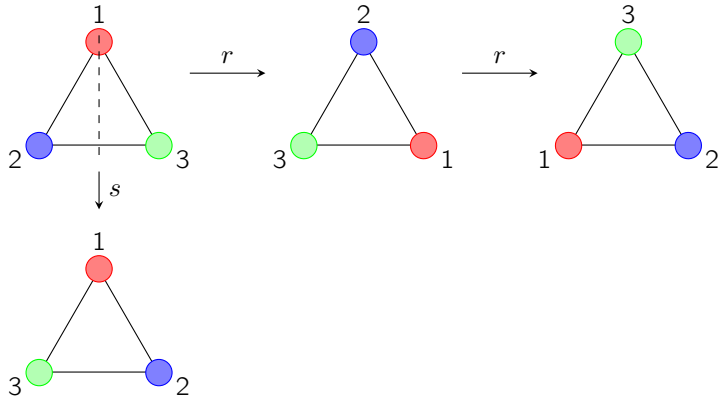

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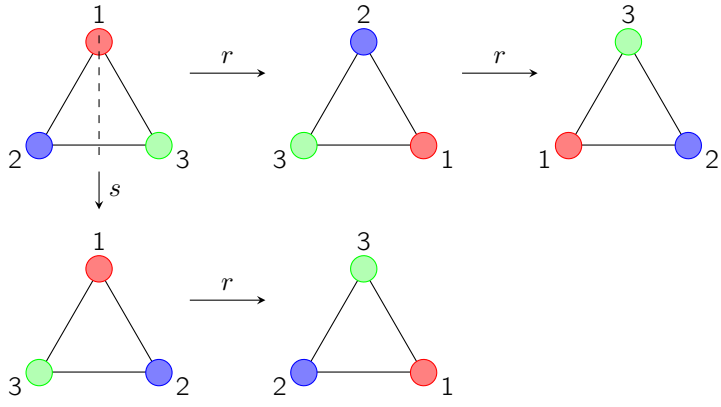

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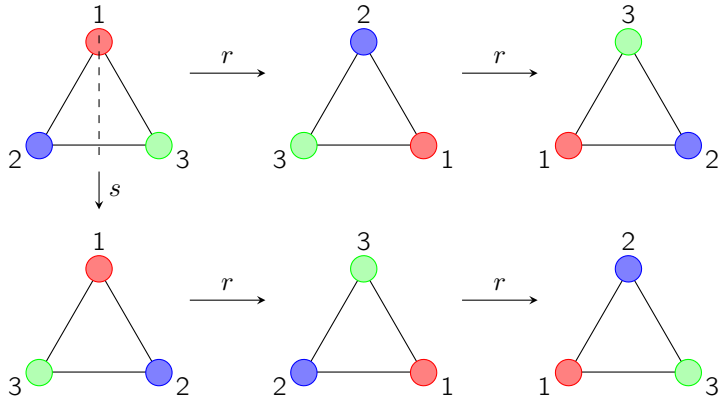

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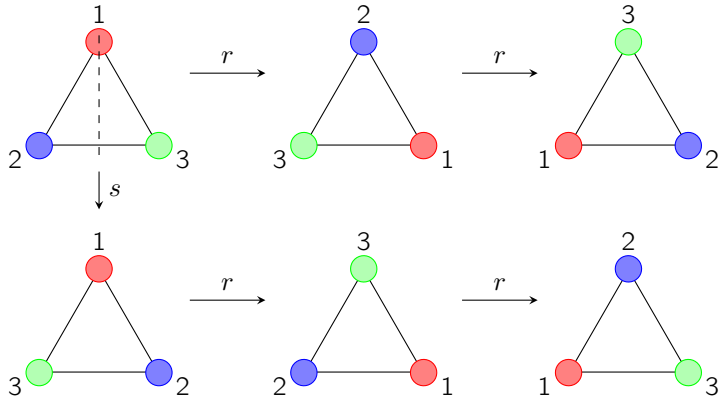
Non-Abelian!



The Dihedral Coset Problem

Given quantum states of the form

$$\frac{1}{\sqrt{2}}|0, x\rangle + \frac{1}{\sqrt{2}}|1, x + s \bmod N\rangle,$$

for $x \leftarrow \mathcal{U}(\mathbb{Z}_N)$, find s.

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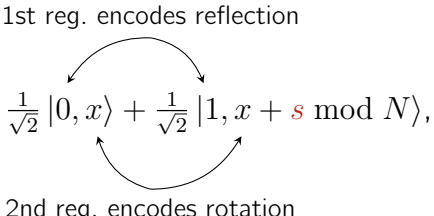

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LWE and the Dihedral Coset Problem

Dimension: n, modulus: $q = \mathrm{poly}(n)$

LWE: Given

$$(\mathbf{a}_1, \langle \mathbf{a}_1, \mathbf{s} \rangle + e_1 \bmod q)$$
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$$(\mathbf{a}_m, \langle \mathbf{a}_m, \mathbf{s} \rangle + e_m \bmod q)$$

with $\|\mathbf{e}\| \ll q$, find $\mathbf{s}$.

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$\leq$ [Regev'02]

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Best known attacks:

BKW / lattices:

$$\exp \mathcal{O}\left(n \cdot \frac{\log q \log n}{(\log q - \log |e_i|)^2} \right)$$

Kuperberg, Regev:

$$\exp \mathcal{O}(\log \ell + \log N / \log \ell)$$

The reduction produces $\ell = \mathrm{poly}(n)$, $N = 2^{n^2}$

Inverse direction

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Answer:

No, but we known that $\underline{\text{E}}$DCP $\leq$ LWE [BKSW18]

Extrapolated DCP

EDCP

for a distribution $\mathcal{D}$

$$\sum_{j \in \mathsf{sup}(\mathcal{D})} \mathcal{D}(j) \, |j\rangle \, |\mathbf{x} + j \cdot \mathbf{s} \bmod q\rangle$$

DCP

$$|0\rangle \, |x\rangle + |1\rangle \, |x + s \bmod N\rangle$$

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Examples:

U-EDCP$_{n,q,M}$

$$\sum_{j=0}^{M-1} \lvert j \rangle \lvert \mathbf{x} + j \cdot \mathbf{s} \bmod q \rangle$$

G-EDCP$_{n,q,r}$

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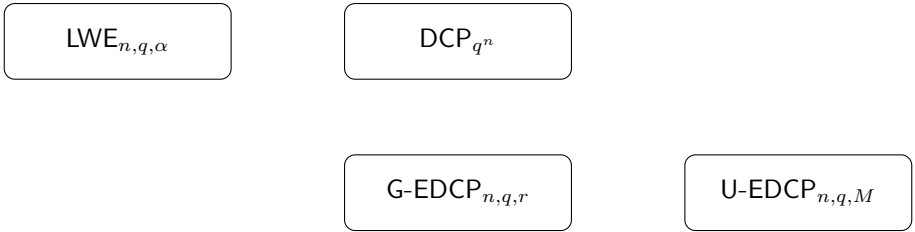
Main result of [BKSW18]:

$$\text{LWE} \iff \text{G-EDCP} \iff \text{U-EDCP} < \text{DCP}$$

Reductions btw LWE, DCP, EDCP



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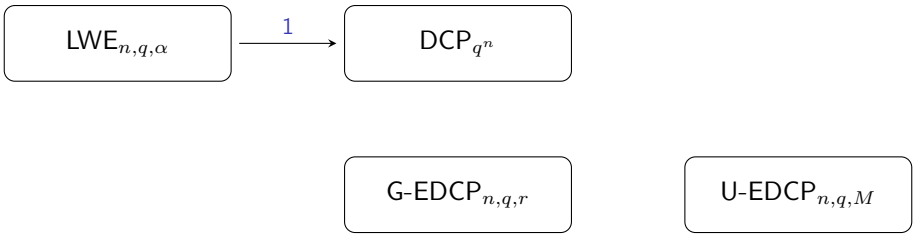

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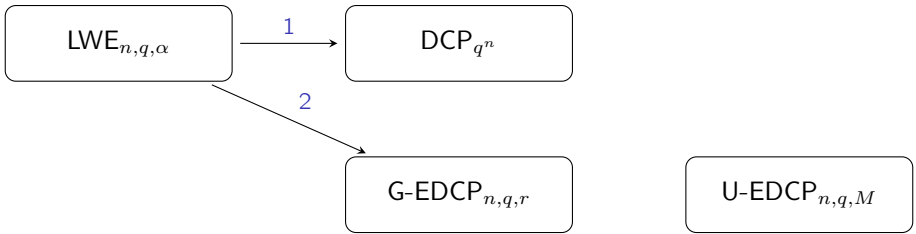

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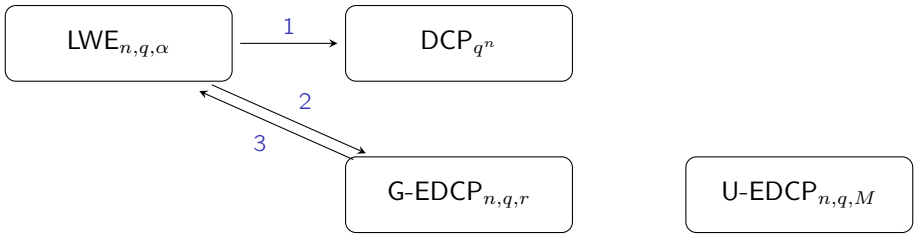

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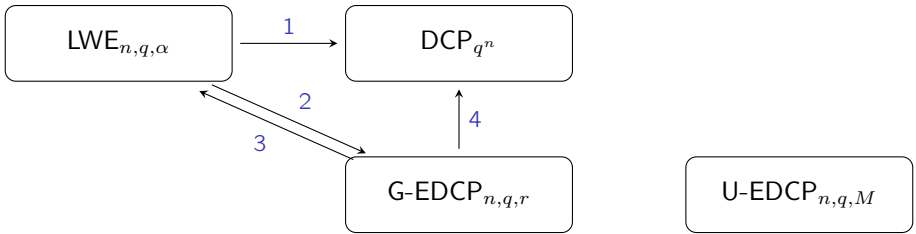

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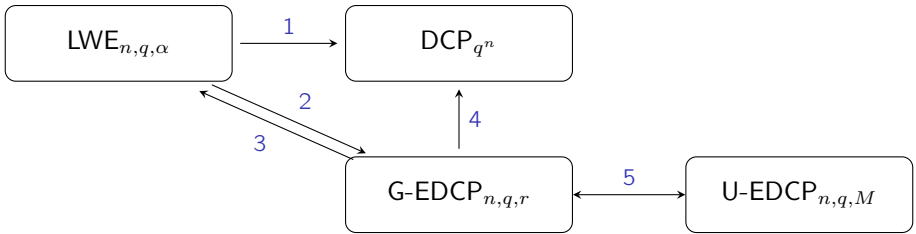

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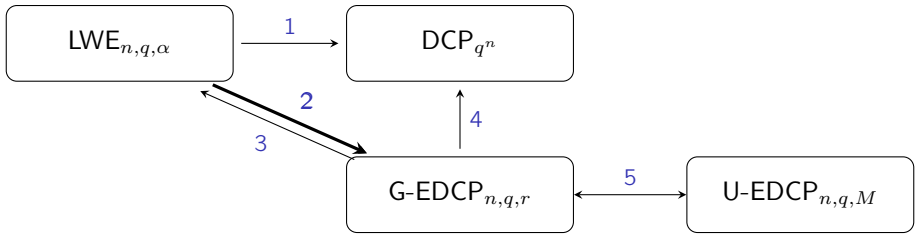

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Apply $\mathbf{A}\mathbf{x} - j\mathbf{b}$ to the third register

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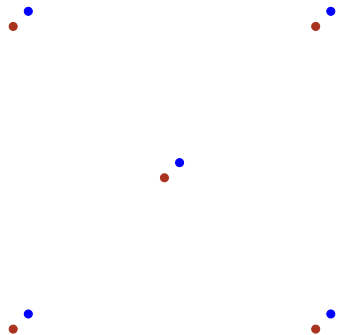

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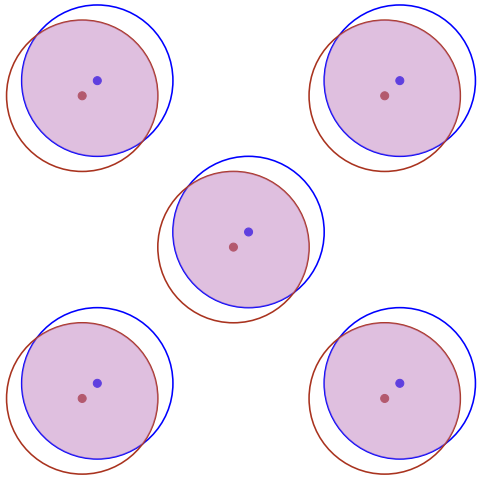

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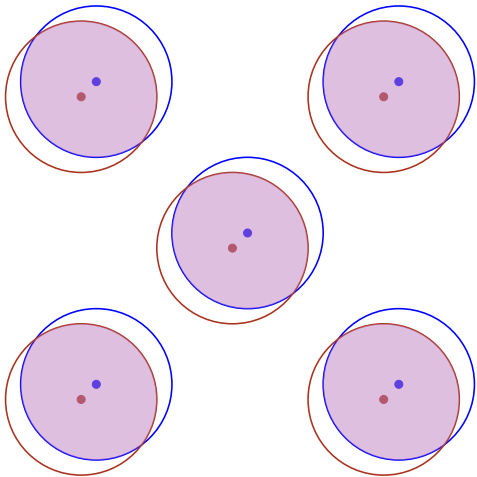

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If we measure a point in the intersection, the resulting state is

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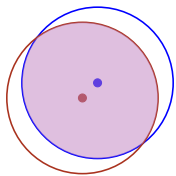

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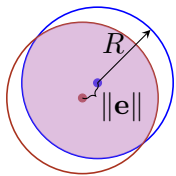

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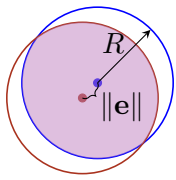

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Take $R\approx\lambda_1(\Lambda_q(\mathbf{A}))\approx q \implies \mathcal{O}(q/(\sqrt{n}\|\mathbf{e}\|))$ 'good' measurements that result in

$$\sum_{j\in\mathbb{Z}} \rho(j)\,|j\rangle\,|\mathbf{x}+j\mathbf{s}\rangle\,|\mathbf{A}\mathbf{x}-j\mathbf{e}\rangle \xrightarrow{\text{0-ize the last register}} \sum_{j\in\mathbb{Z}} \rho(j)\,|j\rangle\,|\mathbf{x}-j\mathbf{s}\rangle$$

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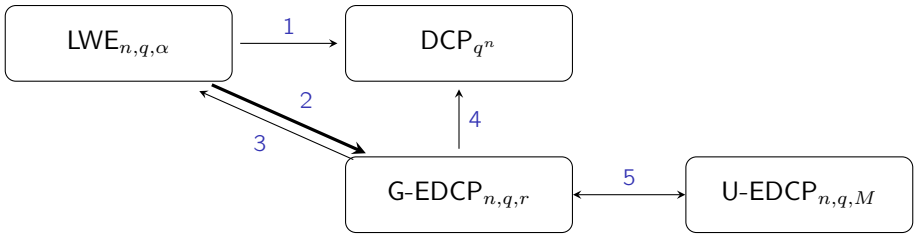

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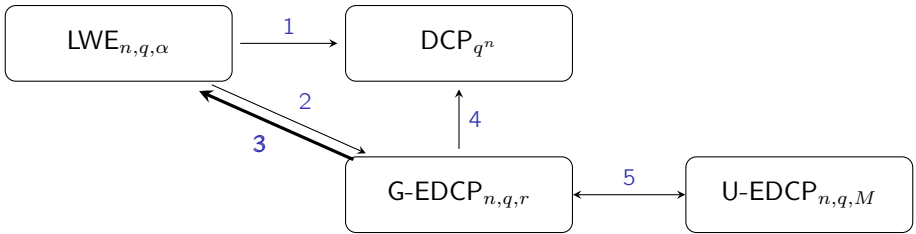
Reductions btw LWE, DCP, EDCP



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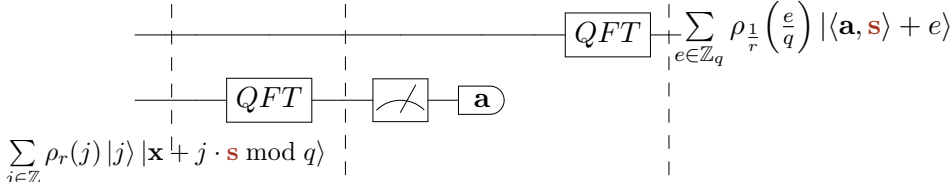
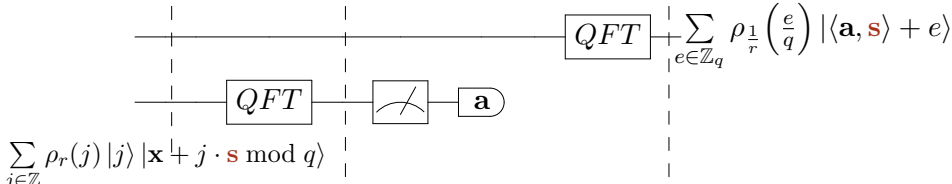
G-EDCP $\leq$ LWE



G-EDCP $\leq$ LWE

$$\omega_q = e^{2\pi i/q}$$



$$(1) : \sum_{\mathbf{a}\in\mathbb{Z}_q^n}\sum_{j\in\mathbb{Z}} \omega_q^{\langle(\mathbf{x}+j\cdot\mathbf{s}),\mathbf{a}\rangle}\cdot\rho_r(j)\,|j\rangle\,|\mathbf{a}\rangle$$

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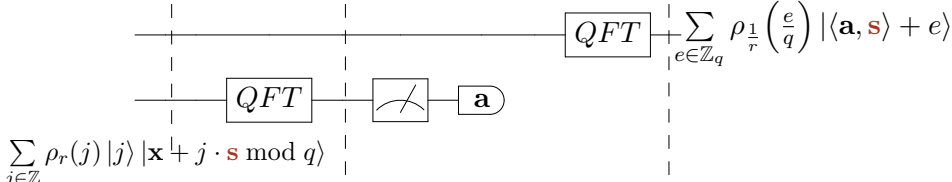

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$$(2): \sum_{b\in\mathbb{Z}_q}\sum_{j\in\mathbb{Z}} \omega_q^{j\cdot(\langle\mathbf{a},\mathbf{s}\rangle+b)}\cdot\rho_r(j)\,|b\rangle$$

G-EDCP $\leq$ LWE

$\omega_q = e^{2\pi i/q}$

Poisson summation (PSF):

$$\rho_r(\mathbb{Z}^n + \mathbf{u}) = r^n \sum_{\mathbf{x}\in\mathbb{Z}^n} e^{2\pi i\langle \mathbf{u},\mathbf{x}\rangle}\rho_{1/r}(\mathbf{u})$$

QFT

$$\sum_{e\in\mathbb{Z}_q} \rho_{\frac{1}{r}}\left(\frac{e}{q}\right)|\langle \mathbf{a},\mathbf{s}\rangle + e\rangle$$

(2)

$$(1): \sum_{\mathbf{a}\in\mathbb{Z}_q^n}\sum_{j\in\mathbb{Z}} \omega_q^{\langle(\mathbf{x}+j\cdot\mathbf{s}),\mathbf{a}\rangle}\cdot\rho_r(j)\,|j\rangle\,|\mathbf{a}\rangle$$

$$(2): \sum_{b\in\mathbb{Z}_q}\sum_{j\in\mathbb{Z}} \omega_q^{j\cdot(\langle \mathbf{a},\mathbf{s}\rangle+b)}\cdot\rho_r(j)\,|b\rangle \xrightarrow{\text{PSF}} \sum_{b\in\mathbb{Z}_q}\sum_{j\in\mathbb{Z}} \rho_{\frac{1}{r}}\left(j+\frac{\langle \mathbf{a},\mathbf{s}\rangle+b}{q}\right)|b\rangle$$

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(2)

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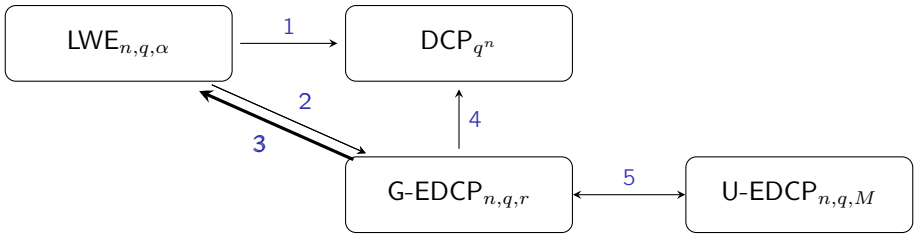
Reductions btw LWE, DCP, EDCP



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Complexity of (U)-EDCP $\sum_{j=0}^{M-1} |j\rangle |\mathbf{x} + j \cdot \mathbf{s} \bmod q\rangle$

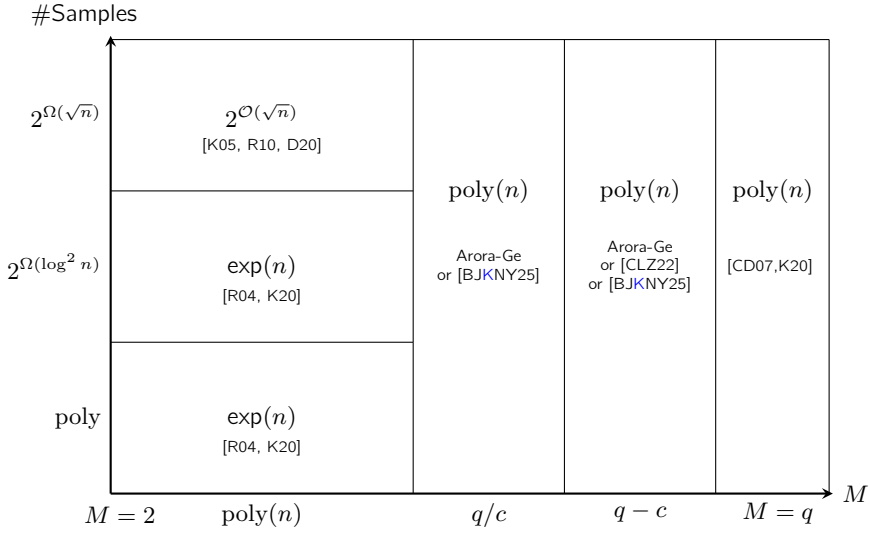

Figure: Complexity of U-EDCP$_{n,q,M}$.

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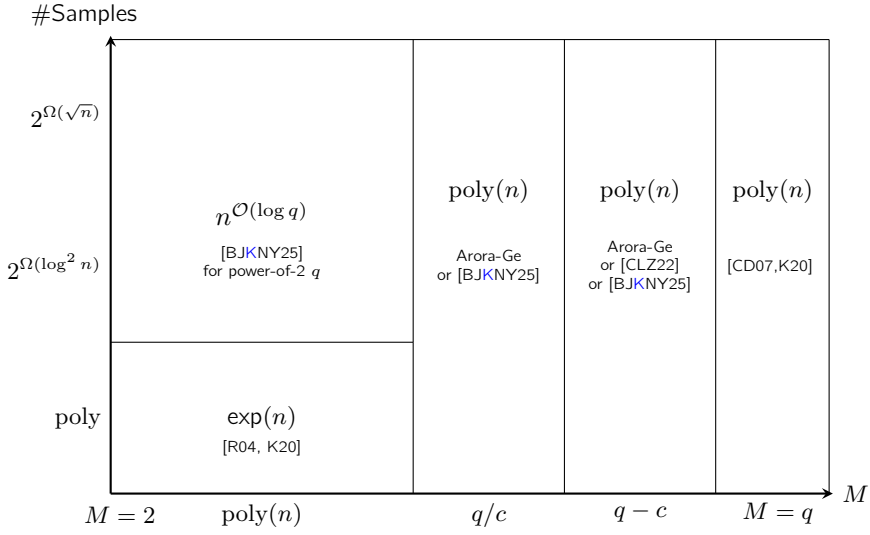


Figure: Complexity of U-EDCP$_{n,q,M}$.

Thoughts

- Having a reduction "LWE $\leq$ EDCP " that produces $n^{\log q}$ EDCP samples will lead to quasi-polynomial quantum algorithm for LWE
- Extending the algorithm from [BJKNY25] to moduli $q = p^t$ for $p = \mathrm{poly}(n)$, you'll get a $\mathrm{poly}(n)$ algorithm for LWE
- A sub-exponential algorithm for EDCP with $\mathrm{poly}(n)$ samples would lead to a sub-exponential attack on LWE
- Recent eprint 2026/155 shows a module-LWE analogue of EDCP. Is it easier than EDCP?

What about codes?

For $\mathbf{A} \in \mathbb{F}_2^{n \times k}, \mathbf{b} = \mathbf{A}\mathbf{s} + \mathbf{e}$, following the same steps as for LWE we can construct:

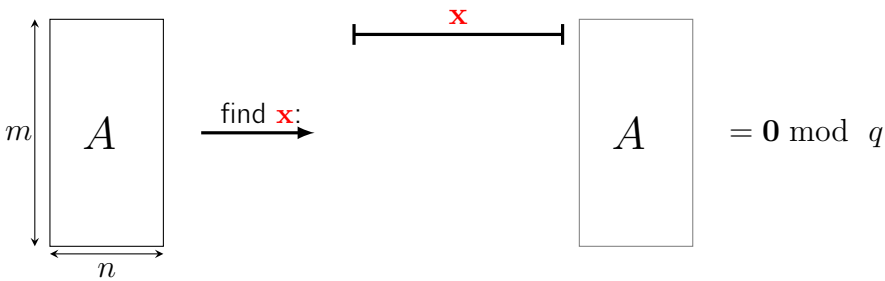
$$\sum_{\mathbf{x} \in \mathbb{F}_2^n} |0, \mathbf{x}, \mathbf{A}\mathbf{x} \bmod 2\rangle + |1, \mathbf{x} + \mathbf{s}, \mathbf{A}\mathbf{x} + \mathbf{e} \bmod 2\rangle$$

"Separating" $(\mathbf{A}\mathbf{x}, \mathbf{A}\mathbf{x} + \mathbf{e})$ from $(\mathbf{A}\mathbf{x}', \mathbf{A}\mathbf{x}' + \mathbf{e})$ is a version of the decoding problem for $\mathbf{A}$.

Part II

Short Integer Solution Quantumly

Definition of SIS

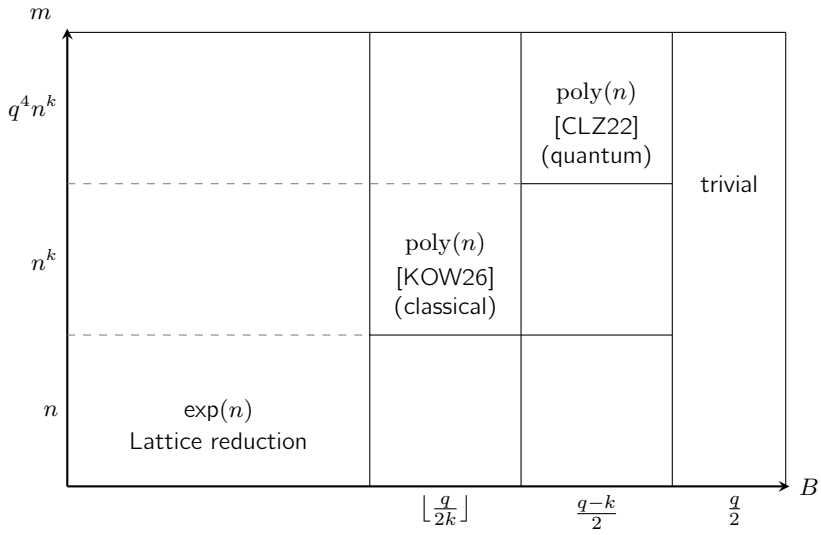


and $0 < \|\mathbf{x}\| \leq B$

Example: to break Dilithium signature, you need to solve SIS for $\|\mathbf{x}\|_\infty < \frac{q}{8}$.

Here we consider $\mod q \in \left[-\frac{q}{2}, \frac{q}{2}\right)$.

SIS_∞ Quantumly/Classically



$k = \Theta(1)$.

Final thoughts and open problems

- Does ball-intersection give the optimal state separation strategy? Consider costlier but more successful function to separate $\mathbf{A}\mathbf{x} + \mathbf{e}_1$ and $\mathbf{A}\mathbf{x} + \mathbf{e}_2$.
- Modulus-dimension switching for LWE allows to reduce LWE to dimension n/k with modulus q^k. If a converse reduction to LWE in dimension $n \log q$ and modulus 2 were possible, we could have solved $\mathsf{EDCP}_{n \log q, 2}$ in polynomial time
- Improve the reduction for binary/small/structured secrets

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